NORWEGIAN OFFSHORE DIRECTORATE

# Evaluating measurement data in connection with measuring fluid flow

## Appendix 3 to the Guidelines for the Measurement Regulations

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# Introduction

This Appendix deals with the principles involved in evaluating measurement data in connection with measuring fluid flow. The purpose of this Appendix is to provide a more detailed explanation of the requirements set for calibration and verification of flow meters. The Appendix is mainly based on ISO 5168 (cf. Appendix 2).

# Measurement error and uncertainty

## Mean value of measurement errors

The mean value of measurement errors is determined at each flow rate as an arithmetic mean determined by the following equation:

$$\bar{E} = \frac{\sum_{i=1}^{n} E_i}{n} \tag{1}$$

where

$E_i$ is the *i*-th relative measurement error,
$\bar{E}$ is the mean relative measurement error, and
$n$ is the number of measurements of the same quantity at a single flow rate.

The *i*-th relative measurement error is calculated using the following equation:

$$E_i = \frac{Q_{ind} - Q_{ref}}{Q_{ref}} \tag{2}$$

where $Q_{ind}$ is the flow rate measured by a meter during tests and $Q_{ref}$ is the flow rate measured with the measurement standard, corrected for any thermodynamic differences in the fluid at the meter during tests and at the measurement standard.

## Random uncertainty of mean value of measurement errors

The random uncertainty of the mean value of measurement errors is determined at each flow rate as a type A uncertainty of a series of $n$ measurements determined by the following equation:

$$U_{AM-E} = \frac{U_{AS-E}}{\sqrt{n}} \tag{3}$$

where $U_{AS-E}$ is random uncertainty in single measurements (repeatability of a meter) which is determined at each flow rate and is calculated using the following equation:

$$U_{AS-E} = t_{95,n-1} \cdot \sqrt{\frac{\sum_{i=1}^{n} (E_i - \bar{E})^2}{n-1}} = t_{95,n-1} \cdot s \tag{4}$$

and where $s$ is the experimental standard deviation (for a series of $n$ measurements of the same quantity), $t_{95,\,n-1}$ is the Student t-distribution factor for a confidence level of 95 % and $n - 1$ degrees of freedom.

It follows from e.g. API MPMS 13.3 that the experimental standard deviation of a series of $n$ measurements can be approximated using the following equation:

$$s \cong \frac{w_{(n)}}{d_{(n)}} \tag{5}$$

where $w_{(n)}$ is the range, the difference between the maximum and minimum values for a set of measurement data, and $d_{(n)}$ is a conversion factor for estimating standard deviations for $n$ measurements. Values for $d_{(n)}$ can e.g. be found in Table E.1 in API MPMS 13.3, they can also be calculated based on the expected values of the ratio $w_{(n)}/s$ for normally distributed random individual measurements. By combining the above equations, an approximation of the uncertainty of the mean value of a series of $n$ measurements can be expressed by the equation:

$$U_{AM-E} \cong \frac{t_{95,n-1} \cdot w_{(n)}}{\sqrt{n} \cdot d_{(n)}} \tag{6}$$

Example: $w_{(5)} = 0{,}05\ \% \rightarrow U_{AM-E} = 0{,}027\ \%$.

## Combined uncertainty of mean value of measurement errors

The combined uncertainty of the mean value of measurement errors is determined at each flow rate using the following equation:

$$U_{CM-E} = \sqrt{{U_{AM-E}}^2 + {U_{CMC}}^2} \tag{7}$$

where $U_{CMC}$ is the combined uncertainty of the calibration setup (CMC is an abbreviation for "Calibration and Measurement Capability"), including the uncertainty of the measurement standard. Since $U_{AM-E}$ can be reduced by increasing the number of measurements in the series, could often $U_{CM-E}$ be marginally greater than $U_{CMC}$.

## Evaluation of measurement errors

It follows from e.g. OIML R137:2012 and ISO 17089:2019 that a measurement error can reasonably be considered to be within an specified error limit (MPE – maximum permissible error) if the mean value of the measurement error is within the acceptance limits in Table 1.

*Table 1. Acceptance limits for measurement errors*

| Mean value of measurement errors (deviation in indication) | Combined uncertainty of mean value | Acceptance limit |
|---|---|---|
| $\bar{\boldsymbol{E}}$ | $U_{CM-E} < \frac{1}{3} \cdot MPE$ | $MPE$ |
| | $U_{CM-E} \in [1/3 \cdot MPE, MPE]$ | $\frac{4}{3} \cdot MPE - U_{CM-E}$ |
| | $U_{CM-E} > MPE$ | Undefined |

If $U_{CM-E} > MPE$, the combined uncertainty of the mean value is too high to verify compliance with the MPE requirement.

The method and principle described above are expanded upon in JCGM 106:2012 and OIML G-19:2017. These documents may be useful in developing methods to evaluate measurement errors and in establishing acceptance limits for calibration and verification (see e.g. Figure 7 in JCGM 106:2012).

Example: Evaluation of deviation in indications from two meters in series

The example below presumes that the methodology described above can be used for double instrumentation where verification of compliance with requirements for instrumental measurement uncertainty takes place by measuring the deviation in indications from two meters. Here, the meter acting as primary meter (Meter A) will typically be the one to be verified and the other meter (Meter B) will serve as reference. At a given flow rate, the *i*-th relative measurement error is determined by the following equation:

$$E_i = \frac{Q_A - Q_B}{Q_A} \tag{8}$$

where $Q_A$ is the flow rate measured by Meter A and $Q_B$ is the flow rate measured by Meter B, corrected for any thermodynamic differences in the fluid at meters A and B. It then follows that the mean value of the measurement error is determined by the following equation:

$$\bar{E} = \frac{\sum_{i=1}^{n} E_i}{n} \tag{9}$$

The combined uncertainty of the mean value of the (relative) measurement error can be determined by the following equation:

$$U_{CM-E} = \sqrt{{U_{AM-E}}^2 + {U_B}^2} \tag{10}$$

where $U_B$ is the instrumental measurement uncertainty of Meter B (value given in uncertainty budget or in certificate), potentially minus uncertainty contributions that are fully correlated between the two meters. It can then be presumed that Meter A satisfies uncertainty limit requirements for instrumental measurement uncertainty, $U_g$, if the mean value of the measurement error is within the acceptance limits in Table 2.

*Table 2. Acceptance limits for deviations in indications from two meters*

| Mean value of measurement errors (deviation in indication) | Combined uncertainty of mean value | Acceptance limit |
|---|---|---|
| $\bar{\boldsymbol{E}}$ | $U_{CM-E} < \frac{1}{3} \cdot U_g$ | $U_g$ |
| | $U_{CM-E} \in [1/3 \cdot U_g, U_g]$ | $\frac{4}{3} \cdot U_g - U_{CM-E}$ |
| | $U_{CM-E} > U_g$ | Undefined |

For example, an uncertainty limit for instrumental measurement uncertainty of $U_g = 0.20\ \%$ and a combined uncertainty in the mean value of measurement errors of $U_{CM-E} = 0.15\ \%$ will yield an acceptance limit of 0.12 % for the mean value of the measurement error.

# Calibration factor (*K* factor) and uncertainty

## Mean value of *K* factor

The mean value of the *K* factor is determined at each flow rate as an arithmetic mean determined by the following equation:

$$\overline{K} = \frac{\sum_{i=1}^{n} K_i}{n} \tag{11}$$

where

$K_i$ is the *i*-th absolute *K* factor,
$\overline{K}$ is the average absolute *K* factor, and
$n$ is the number of measurements of the same quantity at a single flow rate.

## Random uncertainty of mean value of *K* factor

The random uncertainty of the mean value of the *K* factor is determined at each flow rate as a type A uncertainty of a series of $n$ measurements determined by the following equation:

$$U_{AM-K} = \frac{U_{AS-K}}{\sqrt{n}} \tag{12}$$

where $U_{AS-K}$ is random uncertainty in single measurements (repeatability of a meter) determined by the following equation:

$$U_{AS-K} = \frac{t_{95,n-1}}{\overline{K}} \cdot \sqrt{\frac{\sum_{i=1}^{n}(K_i - \overline{K})^2}{n-1}} = t_{95,n-1} \cdot \frac{s}{\overline{K}} \tag{13}$$

An approximation for $U_{AS-K}$ can be found in the substitution $s \cong w_{(n)}/d_{(n)}$ (cf. Equation (5)).

Example: $w_{(5)} = 0.05\ \% \rightarrow U_{AM-K} = 0.027\%$.

## Combined uncertainty of the mean value of the *K* factor

The combined uncertainty of the mean value of the *K* factor is determined at each flow rate using the following equation:

$$U_{CM-K} = \sqrt{{U_{AM-K}}^2 + {U_{CMC}}^2} \tag{14}$$

# Linearity over the flow rate range

Linearity, expressed by (relative) measurement error, is determined as:

$$\bar{E}_{MAX} - \bar{E}_{MIN}$$

where $\bar{E}_{MAX}$ and $\bar{E}_{MIN}$ are the mean values of the highest and lowest measurement error, respectively, over the flow rate range.

Linearity, expressed by the (absolute) *K* factor, is determined as:

$$\frac{\overline{K}_{MAX} - \overline{K}_{MIN}}{\overline{\overline{K}}}$$

where

$$\overline{\overline{K}} = \frac{\sum_{j=1}^{m} \overline{K}_J}{m} \tag{15}$$

and $m$ is the number of measurements of the same quantity over the flow rate range. The calibration factors $\overline{K}_{MAX}$ and $\overline{K}_{MIN}$ are the mean values of the highest and lowest *K* factor, respectively, over the flow rate range.